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Feedforward and feedback optimal control with memory for offshore platforms under irregular wave forces

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ABSTRACT

This paper studies the vibration suppression of actively controlled jacket-type offshore platforms with fixed delay in the control. Based on the wave theory and Morison equation, an exosystem is designed to describe the irregular wave forces. Through a particular transformation, the original delay system is reduced into a non-delay system. Based on the reduced system, the paper develops a feedforward and feedback optimal control law with memory (FFOCLM). The memory terms in FFOCLM compensate the time-delay in control input. The feedforward term of the controller includes the information of the irregular wave forces. The feedback loop incorporates the displacement and velocity of structure into the control law. The FFOCLM is proved to be existent and unique, and able to stabilize the time-delay system. The feasibility and effectiveness of the presented control law is demonstrated by a numerical example of a jacket-type offshore structure.

1. Introduction

In modern world, oil crisis has become a bottle-neck of economy. Then offshore structures, especially the oil and gas production platforms, play a more and more important role. Located in the hostile environment, the platforms undergo continuous vibration due to the external loads such as wind, wave, and earthquake. To prevent fatigue damage and ensure safety and production efficiency, displacement and acceleration of the platforms should be limited. CII offshore platform is located in the South of BoHai Bay. Due to wave force on the platform and structure problem of legs, the max displacement of CII life platform is 4 cm that makes people on the platform fell uncomfortable. After servicing, the platform works well. In the past decades, active control has attracted much interest and many valuable results have been obtained. To suppress the vibration of offshore structures, two consecutive loops were used to design the feedback process [1]. Feedforward and feedback optimal control law was presented to suppress the vibration of the offshore platforms [2,3]. From the view of frequency domain, H_2 optimal control algorithm was applied to control the lateral vibration of the jacket-type platform [4]. Optimal feedforward and feedback H_2 approach was proposed based on the augmented system with wave load [5].

Previous studies have presented many methods to depress the vibration efficiently, but most of them do not consider the effect of time-delay. However, in practical control systems, time-delay exists widely in the data acquisition, on-line data

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processing, computation and application of control force. The structures of civil engineering contain inevitable time-delay in the control action. The application of unsynchronized control forces due to time-delay causes the degradation of the system performance and it may even render the controlled system to be unstable. Then we should pay serious attention to the control delay problem in the vibration suppression of the platforms.

Researchers have presented some approaches to solve the control delay problem [6]. A direct way is to transform the time-delay system into a non-delay system through augmenting state [7,8]. The method can be completed easily, but it may result in an exponential growth of system dimension. Then the computation progress for the high dimension system with long time-delay is complicated. Meanwhile, for discrete time system, state-augmented method cannot always ensure the controllability and observability of the control system. Predictive algorithms were also presented [9,10]. They work well for single degree of freedom (SDOF) systems with short time-delay, but the control efficiency decrease with the increase of time-delay. Classic Smith predictor was presented and used for stable processes with long time-delay [11,12]. For higher value of time-delay, although the stability of the controlled structure is guaranteed by Classic Smith predictor, the performance degrades drastically. Modified Smith predictor has been proposed for the integrating and unstable time-delay system with load disturbance [13]. The phase-shift method [14] is well-known in civil engineering to compensate the time-delay. In the SDOF systems, the method works well. But with the method, one cannot guarantee the stability and performance of the multiple degree of freedom (MDOF) systems. We can see that, in the current literature, the study on the active control with time-delay is not very thorough. It is important and necessary to develop an efficient vibration control law applicable to the civil engineering that can solve the problem due to time-delay in the control.

This paper investigates the vibration control of offshore platforms with active mass damper (AMD) device. Time-delay exists in the control action. The rest of this paper is organized as follows. In Section 2, an exosystem is designed to simulate irregular wave forces. In Section 3, the original control problem of offshore structures is formulated as optimal control for the time-delay discrete system with persistent external disturbance. By a particular transformation, the time-delay system is reduced into delay free system. Section 4 presents the FFOCLM based on reduced system. In Section 5, the validity and performance of the FFOCLM is evaluated by applying it to a numerical example of steel jacket-type offshore platform. FFOCLM is compared against state feedback optimal control law with memory (SFOCLM), the feedforward and feedback optimal control law (FFOCL) that is designed by assuming no time-delay, and predictive optimal control law. We study the sensitivity of damping ratio at the end of Section 5. The last section gives the conclusion of the paper.

2. Irregular wave loads

Offshore structures are exposed to severe environment and wave actions on the platforms cause continuous vibration of structure. For the irregular wave, the elevation $\eta(t)$ can be expressed as

$$\eta(t) = \sum_{j=1}^{r} \eta_j(t) = \sum_{j=1}^{r} A_j \cos\theta_j \tag{1}$$

where t is the time, r is the number of wave components, $A_j = \sqrt{2S_\eta(\omega_j)\Delta\omega_j}$, $\theta_j = -\omega_j t + \varepsilon_j$, S_η is the wave spectrum, ω_j is the frequency of the jth wave component, ε_j is the random phase angle uniformly distributed between 0 and 2π.

Assume that waves come from the direction where the wave force is the strongest, and that is the direction of x-axis.

The following assumptions hold through the paper

Assumption 1.

(1) The wave propagation is unidirectional;
(2) The platform has been idealized as a monopod structure.

According to wave theory, the forces induced by wave components constitute the total wave forces acting on the offshore structures. For the jth wave component, the wave force is

$$f_j(t) = \int_0^d p_j(z,t)\varphi(z)\,\mathrm{d}z \tag{2}$$

where d is the water depth, z is the vertical coordinate axis (zero at the sea bottom), $\varphi(z)$ is the shape function of the structure, $p_j(z,t)$ is the force of the jth wave component acting on per unit length of the cylinder. The force consists of linear inertia force and nonlinear drag force. The drag force brings finite-memory to the system but the contribution to response is limited. Then $p_j(z,t)$ can be calculated by linearized Morison equation

$$p_j(z,d,t) = C_d\rho D\sqrt{8/\pi}\sigma_{v_j} v_j(z,d,t)/2 + C_m\rho\pi D^2 \dot{v}_j(z,d,t)/4, \tag{3}$$

in which C_d is the drag coefficient, C_m is the inertia coefficient, ρ is the fluid density, D is the diameter of the cylinder, v_j and $\dot{v}_j$ are horizontal velocity and acceleration of the water particle in the jth wave component, respectively, σ_{v_j} is the standard deviation of v_j.

$$v_j(z,d,t) = \omega_j \,\mathrm{ch}(k_j z)/\mathrm{sh}(k_j d)\eta_j(t) \triangleq T_{v_j}(\omega_j,k_j,z,d)\eta_j(t), \tag{4}$$

$$\dot{v}_j(z,d,t) = -\omega_j^2 \,\mathrm{ch}(k_j z)/\mathrm{sh}(k_j d)\tan\theta_j\eta_j(t) \triangleq T_{a_j}(\omega_j,k_j,z,d)\eta_j(t). \tag{5}$$

where k_j is the wave number of the jth wave component that is determined by the dispersion relationship

$$\omega_j^2 = \mathrm{g}k_j \tanh(k_j d), \tag{6}$$

in which g is the gravitational acceleration.

Substituting Eqs. (4) and (5) into Eq. (3), wave force Eq. (2) can be rewritten as

$$f_j(t) = \int_0^d [C_d\rho D\sqrt{8/\pi}\sigma_{v_j}T_{v_j}/2 + C_m\rho\pi D^2 T_{a_j}/4]\varphi(z)\,\mathrm{d}z\eta_j(t) \triangleq T_j(\omega_j)\eta_j(t). \tag{7}$$

Therefore we get the total wave force acting on the jacket-type platform

$$\mathbf{f}(t) = \sum_{j=1}^{r} f_j(t) = \sum_{j=1}^{r} T_j(\omega_j)\eta_j(t). \tag{8}$$

In the following, we construct an exosystem to simulate the irregular wave forces. Let $\overline{v}_j = A_j\cos(\theta_j)$ and $\overline{\mathbf{v}}(t) = [\overline{v}_1 \ \cdots \ \overline{v}_r]^{\mathrm{T}}$, and note that

$$\ddot{\overline{v}}_j = -\omega_j^2\overline{v}_j, j = 1,2,\ldots,r, \tag{9}$$

then we get

$$\ddot{\overline{\mathbf{v}}}(t) = \begin{bmatrix} -\omega_1^2 & & \\ & \ddots & \\ & & -\omega_r^2 \end{bmatrix} \overline{\mathbf{v}}(t) \triangleq \mathbf{G_a}\overline{\mathbf{v}}(t). \tag{10}$$

Let $\mathbf{w}(t) = [\overline{\mathbf{v}}^T(t) \ \dot{\overline{\mathbf{v}}}^T(t)]^{\mathrm{T}}$, then

$$\dot{\mathbf{w}}(t) = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ \mathbf{G_a} & \mathbf{0} \end{bmatrix} \mathbf{w}(t) \triangleq \mathbf{G_c}\mathbf{w}(t),$$

$$\overline{\mathbf{v}}(t) = [\mathbf{I} \ \ \mathbf{0}]\mathbf{w}(t), \tag{11}$$

where $\mathbf{I} \in \mathbb{R}^{r\times r}$ is unit matrix, $\mathbf{0} \in \mathbb{R}^{r\times r}$ is zero matrix.

According to the definition of $\eta(t)$, one gets

$$\eta_j(t) = \overline{v}_j(t), j = 1,2,\ldots,r. \tag{12}$$

Then Eq. (8) can be rewritten as

$$\mathbf{f}(t) = [T_1(\omega) \ \ \ldots \ \ T_r(\omega)]\overline{\mathbf{v}}(t) = [T_1(\omega) \ \ \ldots \ \ T_r(\omega)][\mathbf{I} \ \ \mathbf{0}]\mathbf{w}(t) \triangleq \mathbf{F_c}\mathbf{w}(t). \tag{13}$$

Therefore the total irregular wave forces could be generated by the following exosystem

$$\dot{\mathbf{w}}(t) = \mathbf{G_c}\mathbf{w}(t),$$

$$\mathbf{f}(t) = \mathbf{F_c}\mathbf{w}(t),$$

$$\mathbf{w}(0) = [\overline{\mathbf{v}}^{\mathrm{T}}(0) \ \dot{\overline{\mathbf{v}}}^{\mathrm{T}}(0)]^{\mathrm{T}}. \tag{14}$$

Let T denote the sampling period, one can get the discrete-time form of Eq. (14)

$$\mathbf{w}(k+1) = \mathbf{G}\mathbf{w}(k), k = 0,1,2,\ldots,$$

$$\mathbf{f}(k) = \mathbf{F}\mathbf{w}(k),$$

$$\mathbf{w}(0) = [\overline{\mathbf{v}}^{\mathrm{T}}(0) \ \dot{\overline{\mathbf{v}}}^{\mathrm{T}}(0)]^{\mathrm{T}} \tag{15}$$

where $\mathbf{G} = \exp(\mathbf{G_c}T)$, $\mathbf{F} = \mathbf{F_c}$, the eigenvalues $\lambda_i(\mathbf{G})$ of $\mathbf{G}$ satisfy

$$|\lambda_i(\mathbf{G})| = 1, i = 1, 2, \ldots, 2r. \tag{16}$$

3. Problem statement

The paper investigates the active vibration control law applicable to civil engineering. For steel jacket-type offshore platform with AMD device, the time-delay in control input is inevitable. In this section, we first give motion equation of the offshore structure. By application of finite element method, the fixed offshore platform can be modeled as a MDOF system. Because the first mode response contributes most to the dynamical model, the platform can be modeled as a SDOF system for simplicity. In general the approximation is adequate for the purpose of the vibration control. The sketch of combined system, an offshore platform with an AMD device, is shown in Fig. 1.

In the followings, the modal mass, natural frequency, and damping ratio of the simplified SDOF system are denoted by m_1, ω_1, and ξ_1, respectively, and the corresponding modal coordinate that refers to the deck motion of the offshore platform is denoted by x_1. The mass, natural frequency, and damping ratio of the AMD device are denoted by m_2, ω_2, and ξ_2, respectively, and the displacement of the AMD device is denoted by x_2. The control force and irregular wave force are denoted by u and f, respectively. τ is the time-delay in control. From physical analysis, we get the motion of the combined system described by the following coupled differential equations

$$\begin{aligned} \ddot{x}_1(t) = &-(\omega_1^2 + \omega_2^2 m_2/m_1)x_1(t) + (\omega_2^2 m_2/m_1)x_2(t) - 2(\xi_1\omega_1 + \xi_2\omega_2 m_2/m_1)\dot{x}_1(t) + (2\xi_2\omega_2 m_2/m_1)\dot{x}_2(t) \\ &+ 1/m_1(f(t) - u(t-\tau)), \end{aligned}$$

$$\ddot{x}_2(t) = \omega_2^2(x_1(t) - x_2(t)) + 2\xi_2\omega_2(\dot{x}_1(t) - \dot{x}_2(t)) + u(t-\tau)/m_2. \tag{17}$$

By introducing state vector $\mathbf{x} = [x_1 \ \ x_2 \ \ \dot{x}_1 \ \ \dot{x}_2]^{\mathrm{T}}$, one can obtain state-space model for the combined system

$$\dot{\mathbf{x}}(t) = \overline{\mathbf{A}}\mathbf{x}(t) + \overline{\mathbf{B}}\mathbf{u}(t-\tau) + \overline{\mathbf{E}}\,\mathbf{f}(t),$$

$$\mathbf{x}(0) = \varphi, \tag{18}$$

where

$$\overline{\mathbf{A}} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -(\omega_1^2 + \omega_2^2 m_2/m_1) & \omega_2^2 m_2/m_1 & -2(\xi_1\omega_1 + \xi_2\omega_2 m_2/m_1) & 2\xi_2\omega_2 m_2/m_1 \\ \omega^2 & -\omega^2 & 2\xi_2\omega_2 & -2\xi_2\omega_2 \end{bmatrix},$$

$$\overline{\mathbf{B}} = [0 \ \ 0 \ \ -1/m_1 \ \ 1/m_2]^{\mathrm{T}}, \overline{\mathbf{E}} = [0 \ \ 0 \ \ 1/m_1 \ \ 0]^{\mathrm{T}}.$$

Taking T as sampling period and using zero order hold, we get the discrete time form of Eq. (18)

$$\mathbf{x}(k+1) = \mathbf{A}\mathbf{x}(k) + \mathbf{B_1}\mathbf{u}(k-h) + \mathbf{E}\mathbf{f}(k), k = 0, 1, 2, \ldots,$$

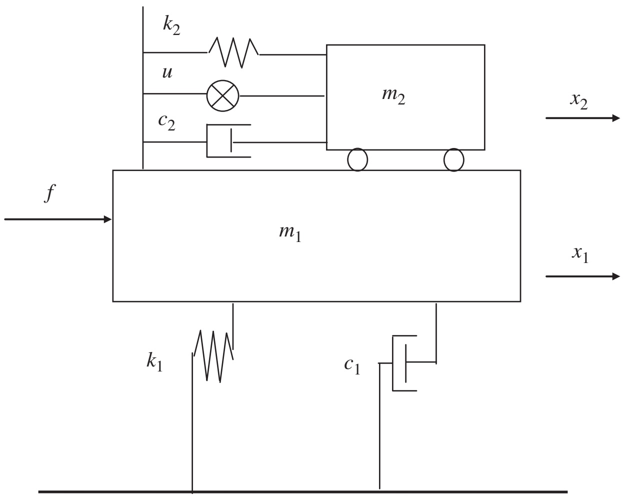

Fig. 1. Sketch of the platform-AMD system.

$$\mathbf{x}(0) = \varphi,$$

$$\mathbf{u}(k) = 0, k = -h, -h+1, \ldots, -1. \tag{19}$$

where $\mathbf{x}(k) \in \mathbb{R}^n$, $\mathbf{u}(k) \in \mathbb{R}^m$, and $\mathbf{f}(k) \in \mathbb{R}^p$ are the state variable, control input, and persistent wave loads, respectively. $h \in \mathbb{N}$ is control delay, $\tau = hT - m\ (0 \leq m < T)$, φ is the initial value of state variable, $\mathbf{A} = \exp(\overline{\mathbf{A}}T)$, $\mathbf{B_1} = \int_0^T \exp(\overline{\mathbf{A}}t)\,\mathrm{d}t\overline{\mathbf{B}}$, $\mathbf{E} = \int_0^T \exp(\overline{\mathbf{A}}t)\,\mathrm{d}t\overline{\mathbf{E}}$.

By following transformation

$$\mathbf{z}(k) = \mathbf{x}(k) + \sum_{i=k-h}^{k-1} \mathbf{A}^{k-i-1}\mathbf{A}^{-h}\mathbf{B_1}\mathbf{u}(i), \tag{20}$$

we reduce the original control delay system Eq. (19) into a delay-free system

$$\mathbf{z}(k+1) = \mathbf{A}\mathbf{z}(k) + \mathbf{B}\mathbf{u}(k) + \mathbf{E}\mathbf{f}(k), k = 1, 2, \ldots,$$

$$\mathbf{z}(0) = \varphi, \tag{21}$$

where $\mathbf{B} = \mathbf{A}^{-h}\mathbf{B_1}$.

Optimal control regulates the response of structure to internal or external excitation by minimizing a given cost index. Then we adopt it to depress the vibration of offshore structures. The active control using optimal control law is prior to passive control. In this work, we will develop an optimal control law to decrease vibration of such offshore structures below dangerous levels, and guarantee the stability and performance of the system with time-delay in control action.

Because disturbance $\mathbf{f}(k)$ exists persistently, state variable $\mathbf{z}(k)$ and control force $\mathbf{u}(k)$ will not converge to zero synchronously. Then we take following quadratic average cost index

$$J = \lim_{N\to\infty} 1/N \sum_{k=1}^{N} [\mathbf{z}^{\mathrm{T}}(k)\mathbf{Q}\mathbf{z}(k) + \mathbf{u}^{\mathrm{T}}(k)\mathbf{R}\mathbf{u}(k)], \tag{22}$$

where $\mathbf{Q} \in \mathbb{R}^{n\times n}$ is a positive semi-definite matrix, $\mathbf{R} \in \mathbb{R}^{m\times m}$ is a positive definite matrix.

Assumption 2. $(\mathbf{A}, \mathbf{B_1})$ is completely controllable and $(\mathbf{A}, \mathbf{C})$ is completely observable, where $\mathbf{C}$ is defined by $\mathbf{Q} = \mathbf{C}^{\mathrm{T}}\mathbf{C}$.

It is shown in [15] that $(\mathbf{A}, \mathbf{B_1})$ is completely controllable if and only if $(\mathbf{A}, \mathbf{B})$ is completely controllable. Then the reduced system Eq. (21) is completely controllable according to Assumption 2.

The objective of the presented paper is to find the optimal control $\mathbf{u}^*$ to minimize cost index Eq. (22) subjected to constraints Eqs. (15) and (21).

4. Design of FFOCLM

In this section, we develop a FFOCLM to decrease the vibration of platforms.

Theorem 1. *Consider the optimal control problem described by the system Eqs.* (15), (21), *and the quadratic cost index Eq.* (22). *Under Assumption 2, there exists the following unique stabilizing control law FFOCLM*

$$\mathbf{u}^*_{\mathbf{FFOCLM}}(k) = -\mathbf{S}^{-1}\mathbf{B}^{\mathrm{T}}t[\mathbf{PAx}(k) + \mathbf{PA}^{-h}\sum_{i=k-h}^{k-1}\mathbf{A}^{k-i}\mathbf{B_1}\mathbf{u}^*_{\mathbf{FFOCLM}}(i) + \mathbf{PEf}(k) + \overline{\mathbf{P}}\mathbf{Gw}(k)t], k = 0, 1, \ldots \tag{23}$$

where $\mathbf{S} = \mathbf{R} + \mathbf{B}^{\mathrm{T}}\mathbf{PB}$, $\mathbf{P}$ *is the unique positive definite solution to the following discrete Riccati equation*

$$\mathbf{A}^{\mathrm{T}}\mathbf{P}(\mathbf{I} - \mathbf{BS}^{-1}\mathbf{B}^{\mathrm{T}}\mathbf{P})\mathbf{A} - \mathbf{P} + \mathbf{Q} = \mathbf{0}, \tag{24}$$

$\overline{\mathbf{P}}$ *is the unique solution to the following Sylvester equation*

$$\mathbf{A}^{\mathrm{T}}(\mathbf{I} - \mathbf{PBS}^{-1}\mathbf{B}^{\mathrm{T}})\overline{\mathbf{P}}\mathbf{G} - \overline{\mathbf{P}} = -\mathbf{A}^{\mathrm{T}}(\mathbf{I} - \mathbf{PBS}^{-1}\mathbf{B}^{\mathrm{T}})\mathbf{PEF}. \tag{25}$$

Proof. Applying the maximum principle to system Eq. (21) and quadratic cost index Eq. (22), we get optimal control law

$$\mathbf{u}(k) = -\mathbf{R}^{-1}\mathbf{B}^{\mathrm{T}}\lambda(k+1), k = 0, 1, 2, \ldots, \tag{26}$$

where $\lambda(k)$ is the solution to the following two-point boundary value (TPBV) problem

$$\mathbf{z}(k+1) = \mathbf{A}\mathbf{z}(k) - \mathbf{B}\mathbf{R}^{-1}\mathbf{B}^{\mathrm{T}}\lambda(k+1) + \mathbf{E}\mathbf{f}(k),$$
$$\lambda(k) = \mathbf{Q}\mathbf{z}(k) + \mathbf{A}^{\mathrm{T}}\lambda(k+1), k = 0, 1, 2, \ldots,$$
$$\mathbf{z}(0) = \varphi, \lambda(\infty) = \mathbf{0}. \tag{27}$$

To solve the TPBV problem Eq. (27), let

$$\lambda(k) = \mathbf{P}\mathbf{z}(k) + \overline{\mathbf{P}}\mathbf{w}(k), k = 0, 1, 2, \ldots. \tag{28}$$

From Eqs. (15), (27), and (28), we get

$$\mathbf{R}^{-1}\mathbf{B}^{\mathrm{T}}\lambda(k+1) = \mathbf{S}^{-1}\mathbf{B}^{\mathrm{T}}[\mathbf{PAz}(k) + \mathbf{PEf}(k) + \overline{\mathbf{P}}\mathbf{Gw}(k)]. \tag{29}$$

Then the optimal control law Eq. (26) can be rewritten as

$$\mathbf{u}^*(k) = -\mathbf{S}^{-1}\mathbf{B}^{\mathrm{T}}[\mathbf{PAz}(k) + \mathbf{PEf}(k) + \overline{\mathbf{P}}\mathbf{Gw}(k)], k = 0, 1, 2, \ldots. \tag{30}$$

Substituting Eq. (20) into Eq. (30), we get the optimal control law Eq. (23). Noting Eq. (15), and substituting Eqs. (21), (28), and (30) into the second equation of Eq. (27), we get

$$\begin{aligned}\lambda(k) &= \mathbf{Q}\mathbf{z}(k) + \mathbf{A}^{\mathrm{T}}\lambda(k+1) = \mathbf{Q}\mathbf{z}(k) + \mathbf{A}^{\mathrm{T}}[\mathbf{P}\mathbf{z}(k+1) + \overline{\mathbf{P}}\mathbf{w}(k+1)] \\ &= [\mathbf{Q} + \mathbf{A}^{\mathrm{T}}(\mathbf{I} - \mathbf{PBS}^{-1}\mathbf{B}^{\mathrm{T}})\mathbf{PA}]\mathbf{z}(k) + \mathbf{A}^{\mathrm{T}}(\mathbf{I} - \mathbf{PBS}^{-1}\mathbf{B}^{\mathrm{T}})(\mathbf{PEF} + \overline{\mathbf{P}}\mathbf{G})\mathbf{w}(k).\end{aligned} \tag{31}$$

Comparing the coefficients of Eqs. (28) and (31), we obtain Eqs. (24) and (25).

In the followings, we will prove that the optimal control law Eq. (23) is existent and unique, and is a stabilizing method. Obviously, the existence and uniqueness of Eq. (23) is equivalent to that of $\mathbf{P}$ and $\overline{\mathbf{P}}$. Directly following from Assumption 2, the matrix $\mathbf{P}$ is existent and unique. According to optimal regulator theory,

$$|\lambda_i[(\mathbf{I} - \mathbf{BS}^{-1}\mathbf{B}^{\mathrm{T}}\mathbf{P})\mathbf{A}]| < 1, i = 1, 2, \ldots, n. \tag{32}$$

Noting Eq. (16), we obtain

$$|\lambda_i[\mathbf{A}^{\mathrm{T}}(\mathbf{I} - \mathbf{PBS}^{-1}\mathbf{B}^{\mathrm{T}})]\lambda_j(\mathbf{G})| < 1, i = 1, 2, \ldots, n; j = 1, 2, \ldots, 2r. \tag{33}$$

Then $\overline{\mathbf{P}}$, the solution to Eq. (25), is existent and unique [3,16]. When $\mathbf{P}$ and $\overline{\mathbf{P}}$ are derived, $\mathbf{u}^*$ can be determined from Eq. (23). Therefore the optimal control law is existent and unique.

From Eqs. (20) and (30), one gets

$$\begin{aligned}\|\mathbf{x}(k)\| \le \|\mathbf{z}(k)\| &+ h \max_{1\le s\le h} \|\mathbf{A}^{s-1-h}\| \, \|\mathbf{B_1}\mathbf{S}^{-1}\mathbf{B}^{\mathrm{T}}\mathbf{PA}\| \max_{1\le s\le h} \|\mathbf{z}(k-s)\| \\ &+ h \max_{1\le s\le h} \|\mathbf{A}^{s-1-h}\| \, \|\mathbf{B_1}\mathbf{S}^{-1}\mathbf{B}^{\mathrm{T}}(\mathbf{PDF}+\overline{\mathbf{P}}\mathbf{G})\| \max_{1\le s\le h} \|\mathbf{w}(k-s)\|.\end{aligned} \tag{34}$$

Because $\|\mathbf{z}(k)\|$ and $\|\mathbf{w}(k)\|$ are bounded, $\|\mathbf{x}(k)\|$ is bounded. Then FFOCLM Eq. (23) is a stabilizing control law for time-delay system Eq. (19). This completes the proof. □

5. Simulation result and discussion

In this section, we investigate the feasibility and effectiveness of the presented FFOCLM. The jacket-type offshore platform we study is located in Bohai Bay. The structural parameters are listed as: the total height of platform $L = 41.1$ m, the equivalent characteristic diameter of the platform legs $D = 1.7$ m, the first modal mass $m_1 = 2,371,100$ kg, the natural frequency of platform $\omega_1 = 2.20$ rad/s, the structural damping ratio $\xi_1 = 4$ percent, and the shape function of first mode $\varphi(s) = 1 - \cos(\pi s/(2L))$, $0 \le s \le L$. As Fig. 1 shown, an AMD device is installed on the deck of the platform. The properties of

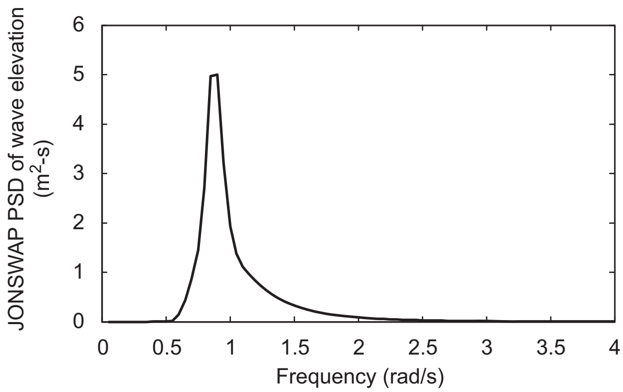

Fig. 2. PSD of wave elevation.

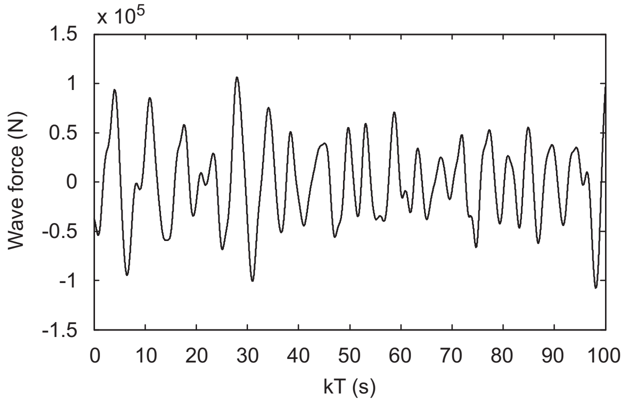

Fig. 3. Wave force acting on the offshore structure.

the AMD device are given as: the mass $m_2 = 11,855\,\text{kg}$, the natural frequency $\omega_2 = 2.33\,\text{rad/s}$, and the structural damping ratio $\xi_2 = 9.32$ percent.

In present study, the irregular wave is expressed by JONSWAP spectrum

$$S_\eta(\omega) = [5H_s^2/(16\omega_0)](\omega_0/\omega)^5 \exp[-1.25(\omega/\omega_0)^{-4}]\gamma^\beta, \tag{35}$$

where H_s is the significant wave height, ω_0 is the peak frequency, ω is the wave frequency, γ is the peakedness parameter, $\beta = \exp[-(\omega - \omega_0)^2/(2\sigma^2\omega_0^2)]$, in which σ is the shape parameter, $\sigma = 0.07\ (\omega \leq \omega_0)$, $\sigma = 0.09\ (\omega > \omega_0)$. Here $H_s = 4\,\text{m}$, $\omega_0 = 0.87\,\text{rad/s}$, and γ=3.3. The power spectrum density (PSD) of wave elevation is shown in Fig. 2. The wave forces f can be calculated from (15) with the water depth d=13.2 m, the drag coefficient C_d=1.2, and the inertial coefficient C_m=2.0. The irregular wave force acting on the structure is displayed in Fig. 3.

The sampling period is T=0.01 s, and the time-delay τ in control input ranges between 0.01 s and 1 s. $\mathbf{Q} = diag(10^7\ \ 0\ \ 10^7\ \ 0)$, $\mathbf{R} = 10^{-6}$.

In the followings, we first compare the FFOCLM with FFOCL, SFOCLM, and predictive optimal control law. After that, we make a sensitivity study on the damping radio.

FFOCL is a feedforward and feedback optimal control law designed without taking time-delay into account, and it can be expressed as follows

$$\mathbf{u}^*_{\mathbf{FFOCL}}(k) = -\mathbf{S_1}^{-1}\mathbf{B_1}^{\mathrm{T}}[\mathbf{P_1Ax}(k) + \mathbf{P_1Ef}(k) + \overline{\mathbf{P}}_\mathbf{1}\mathbf{Gw}(k)], k = 0, 1, 2, \ldots, \tag{36}$$

where $\mathbf{S_1} = \mathbf{R} + \mathbf{B_1^{\mathrm{T}}P_1B_1}$, $\mathbf{P_1}$ is the unique positive definite solution to the following discrete *Riccati* equation

$$\mathbf{A}^{\mathrm{T}}\mathbf{P_1}(\mathbf{I} - \mathbf{B_1S_1}^{-1}\mathbf{B_1^{\mathrm{T}}P_1})\mathbf{A} - \mathbf{P_1} + \mathbf{Q} = \mathbf{0}, \tag{37}$$

$\overline{\mathbf{P}}_\mathbf{1}$ is the unique solution to the following *Sylvester* equation

$$\mathbf{A}^{\mathrm{T}}(\mathbf{I} - \mathbf{P_1B_1S_1}^{-1}\mathbf{B_1^{\mathrm{T}}})\overline{\mathbf{P}}_\mathbf{1}\mathbf{G} - \overline{\mathbf{P}}_\mathbf{1} = -\mathbf{A}^{\mathrm{T}}(\mathbf{I} - \mathbf{P_1B_1S_1}^{-1}\mathbf{B_1^{\mathrm{T}}})\mathbf{P_1EF}, \tag{38}$$

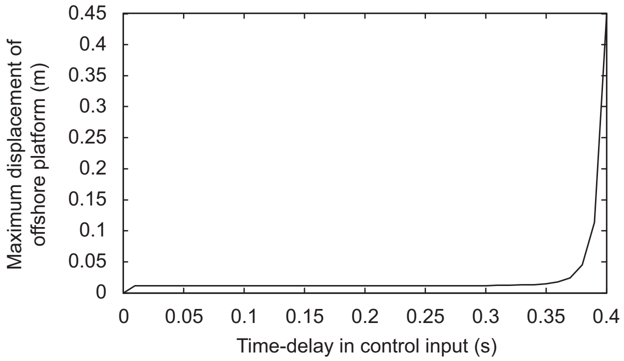

Fig. 4. Maximum displacement of offshore structure with FFOCL varying with time-delays.

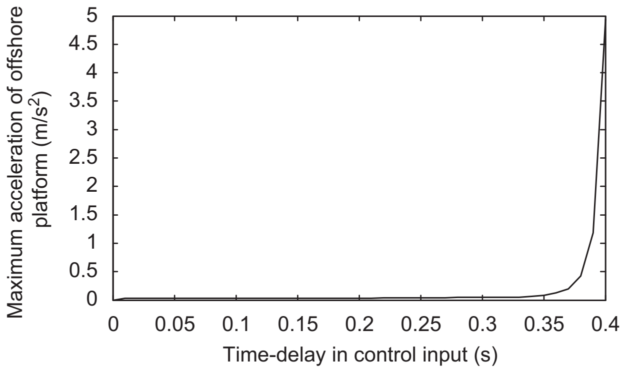

Fig. 5. Maximum acceleration of offshore structure with FFOCL varying with time-delays.

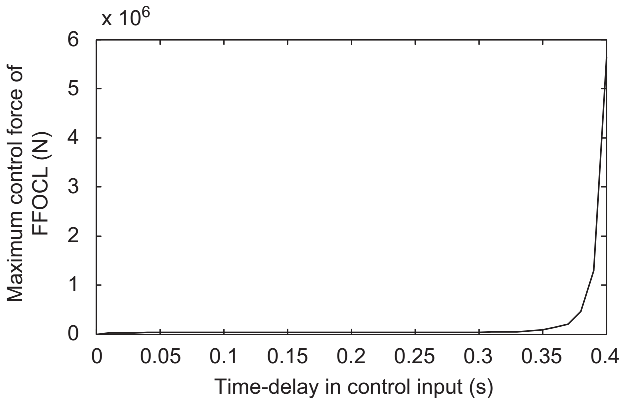

Fig. 6. Maximum control force of FFOCL varying with time-delays.

SFOCLM is the state feedback optimal control law with memory, based on reduced system Eq. (21)

$$\mathbf{u}^{*}_{\mathbf{SFOCLM}}(k) = -\mathbf{S}^{-1}\mathbf{B}^{\mathrm{T}}[\mathbf{PAx}(k) + \mathbf{PA}^{-h}\sum_{i=k-h}^{k-1}\mathbf{A}^{k-i}\mathbf{B_1}\mathbf{u}^{*}_{\mathbf{SFOCLM}}(i)], k = 0, 1, 2, \ldots. \tag{39}$$

The displacement and velocity feedback terms are the common terms of FFOCL, SFOCLM, and FFOCLM. FFOCL and FFOCLM contain same feedforward terms that do not appear in SFOCLM. Compared with FFOCL, we can see that SFOCLM and FFOCLM add finite number of past control actions to the current control input as memory.

When the control law is FFOCL, the maximum displacement and acceleration of the offshore structure with different time-delays are presented in Figs. 4 and 5, respectively. Figs. 6 and 7 show the corresponding maximum control force and the cost index value of the system.

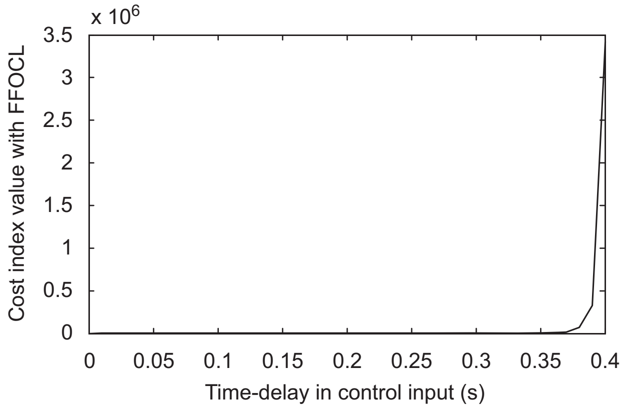

Fig. 7. Cost index value of FFOCL varying with time-delays.

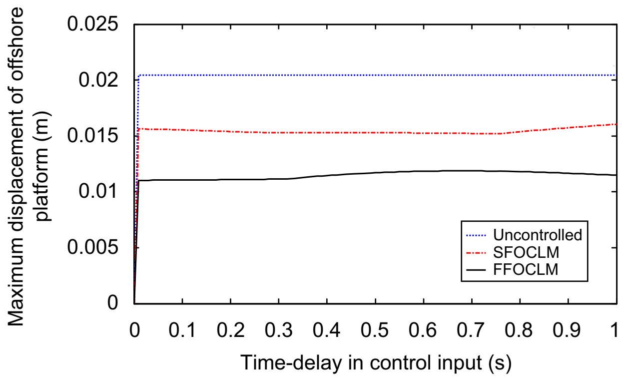

Fig. 8. Maximum displacement of offshore structure with various time-delays.

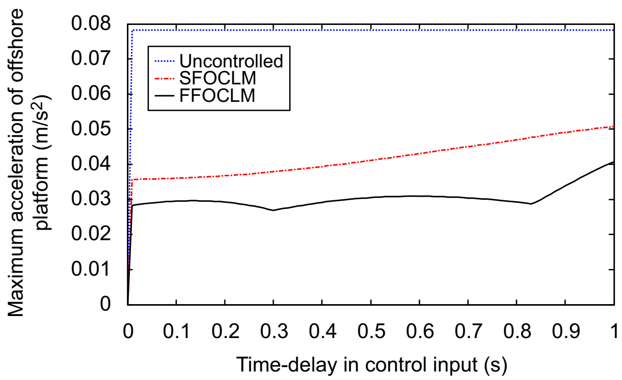

Fig. 9. Maximum acceleration of offshore structure with various time-delays.

When the time-delay is small, the difference of structure response and control force among FFOCL, SFOCLM and FFOCLM is not much. But FFOCL does not consider the effect of time-delay, and then it cannot always guarantee the stability of the system. When $\tau = 0.4\,\mathrm{s}$, the original time-delay system with FFOCL becomes unstable while maintains stable with SFOCLM and FFOCLM.

Next we compare the control laws with memory: SFOCLM and FFOCLM. First, we give the maximum displacement and acceleration of the offshore platform with various time-delays in Figs. 8 and 9. The maximum control forces of different control laws and corresponding cost index values vs. time-delay are shown in Figs. 10 and 11. The system performance with

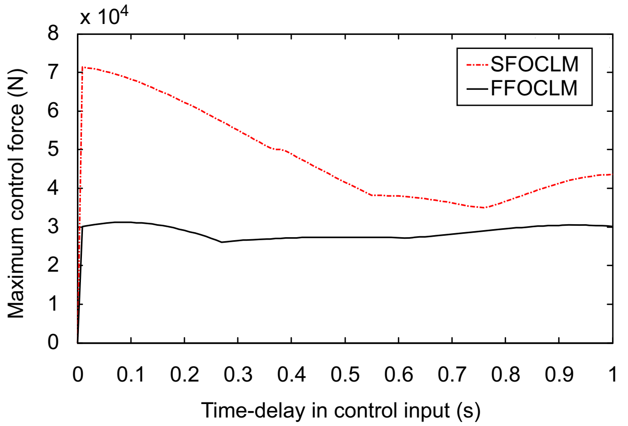

Fig. 10. Maximum control force of offshore structure with various time-delays.

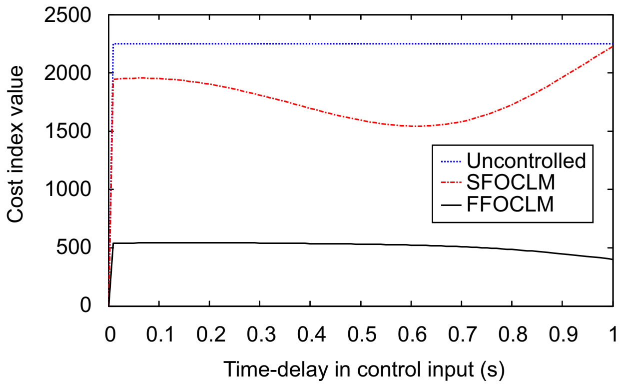

Fig. 11. Cost index value of system with various time-delays.

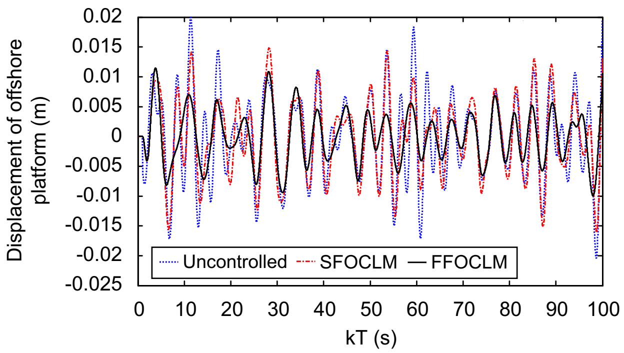

Fig. 12. Displacement of offshore structure with time-delay τ=1 s.

SFOCLM and FFOCLM is much better than the case without control. Compared with SFOCLM, FFOCLM can reduce the vibration more significantly with smaller control force and cost index value. From the simulation results, we can see that FFOCLM can guarantee the performance and the stability of the system with large time-delay. In this study, the time-delay in control is up to 1 s that is long enough in the civil engineering.

For a long time-delay $\tau = 1$ s, we give the displacement and acceleration curves of the offshore system without control, with SFOCLM, and with FFOCLM in Figs. 12 and 13, respectively. The corresponding control curves are presented in Fig. 14. Let J_1 denote the system cost index value with SFOCLM, and J_2 denote that with FFOCLM. We obtain J_1=2228.3, J_2=400.9. FFOCLM can decrease the cost index 82 percent compared with SFOCLM. The displacement, velocity, and acceleration curves show that FFOCLM can reduce the vibration of the offshore structure efficiently. At the same time, the control force of the FFOCLM is smaller than that of SFOCLM as shown in Fig. 14. FFOCLM can reduce the vibration induced by wave force and decrease the effect on the stability of the time-delay in control input.

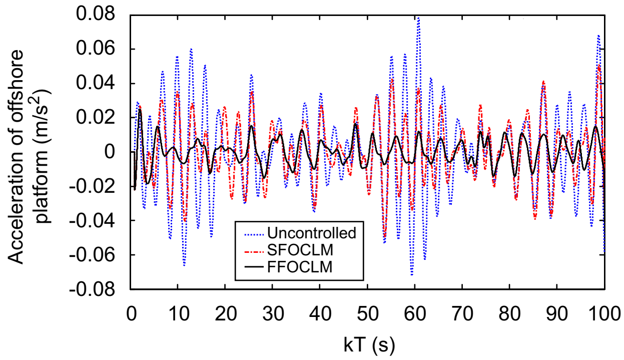

Fig. 13. Acceleration of offshore structure with time-delay τ=1 s.

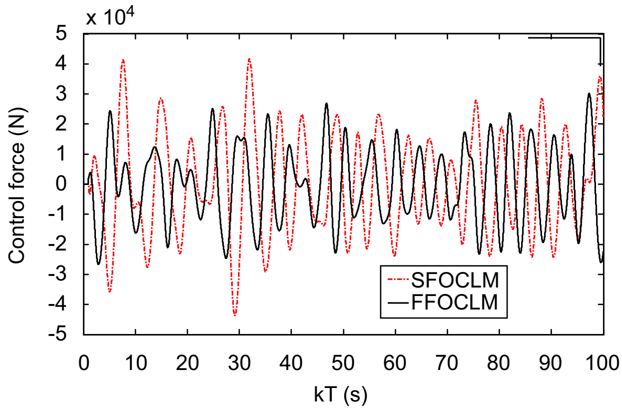

Fig. 14. Control force of offshore structure with time-delay τ=1 s.

Table 1
Maximum displacement of the offshore structure (m).

ξ_2	0.02	0.04	0.05	0.06	0.08	0.09
No contr.	0.0199	0.0203	0.0204	0.0204	0.0204	0.0204
SFOCLM	0.0152	0.0153	0.0153	0.0153	0.0153	0.0154
FFOCLM	0.0115	0.0116	0.0116	0.0117	0.0118	0.0118

Table 2
Maximum acceleration of the offshore structure (m^2/s).

ξ_2	0.02	0.04	0.05	0.06	0.08	0.09
No contr.	0.0772	0.0779	0.0780	0.0780	0.0781	0.0782
SFOCLM	0.0464	0.0466	0.0466	0.0467	0.0468	0.0469
FFOCLM	0.0296	0.0295	0.0294	0.0294	0.0293	0.0293

Table 3
Maximum control of the control law (10^4 N).

ξ_2	0.02	0.04	0.05	0.06	0.08	0.09
SFOCLM	3.4951	3.5213	3.5398	3.5619	3.6167	3.6490
FFOCLM	2.9830	2.9658	2.9596	2.9552	2.9510	2.9511

Table 4
Cost index value of the system.

ξ_2	0.02	0.04	0.05	0.06	0.08	0.09
No contr.	2283.2	2262.1	2255.4	2251.2	2249.0	2250.1
SFOCLM	1719.9	1723.6	1725.0	1726.0	1727.1	1727.0
FFOCLM	467.7	471.0	473.0	475.4	480.9	484.0

For the same offshore structure model, predictive time-delay compensation method is studied in [10], with the sea state $H_s = 2.5\,\text{m}$, $\omega_0 = 1.4\,\text{rad/s}$. Based on prediction of wave force and structural response, the optimal prediction of active control force is used to perform time-delay compensation. The simulation results indicate that the optimal prediction time-delay compensation method is effective for the definite interval of time-delay, that is $\tau \leq 0.2\,\text{s}$. With the increase of time-delay, the prediction cannot keep high precision, and then effect of time-delay compensation control in [10] becomes worth. When time-delay is 0.3 s, compared with the case without control, the standard variation of displacement of structure can be reduced 45.82 percent, velocity 46.91 percent, and acceleration 44.14 percent. The standard variation of the control force is 61.13 kN. With same seastate and structure parameters as above mentioned, the FFOCLM presented in this paper can guarantee the performance and stability and reduce the vibration with the time-delay up to 1 s. When time-delay is 0.3 s and the system is controlled by FFOCLM, the standard variation of platform displacement can be reduced 60.65 percent, velocity 70.29 percent, and acceleration 78.08 percent. The standard variation of control force is 13.54 kN.

In the followings, we make a sensitivity study on the damping ratio. Set the time-delay 0.8 s. When damping ratio of the AMD ξ_2 ranges from 0.02 to 0.09, and the damping ratio of the offshore structure ξ_1 remains 0.04, we get the following results in Tables 1–4.

When there is no control, and controlled by SFOCLM and FFOCLM, with the increase of the damping ratio of AMD device ξ_2, there are changes in the maximum displacement and acceleration of offshore structure, the control force and the cost index value of the system. We can see that damping ratio of the AMD device ξ_2 can affect system performance, but the effect is not very obvious.

The simulation results show that FFOCLM can reduce the wave-induced vibration and compensate the time-delay. From the application's point of view, FFOCLM can ensure the safety and the production efficiency.

6. Conclusions

In this paper, we develop an optimal vibration control law for the jacket-type offshore platforms under irregular wave forces. Time-delay exists in control and affects the stability and performance of system. To simulate the random wave loads, we design an exosystem based on wave theory and the Morison equation, which is easy to complete. The design of FFOCLM considers effect of time-delay and wave force acting on the structure. The memory of the finite past control actions in the FFOCLM compensates the time-delay and the feedforward terms reduce the vibration induced by wave forces. The feedback loop of FFOCLM consists of displacement and velocity. The FFOCLM is proved to be existent and unique, and can guarantee the stability of the time-delay systems.

To demonstrate the effectiveness of the presented control law, a numerical example of a steel jacket-type offshore platform located in Bohai Sea is studied. Simulation results show that the FFOCLM is more efficient and robust than FFOCL, SFOCLM, and optimal predictive algorithm. For the control system with FFOCLM, damping ratio of the AMD device can affect system performance, but the effect is not very obvious. FFOCLM can compensate the time-delay in control input and depress wave-induced vibration efficiently, so the performance and stability of the system can be guaranteed.

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